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Systematic Study of Horizontal Gauge Theories

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ABSTRACT

We analyze all the possible continuous horizontal gauge groups G_H in relation with their possibility to explain $m_b << m_t$. We assume that the only effective fermionic degrees of freedom correspond to the known fermions but allow the possibility of adding a right handed neutrino to each family. We assume that the Higgs fields which generate masses for these fermions, trough renormalizable Yukawa couplings, transform as an irreducible representation of $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_H$. Under these assumptions we find two $U(1)_H$ or $U(1)_{H1} \otimes U(1)_{H2}$ models free of anomalies and able to guarantee that only the top has a renormalizable mass-generating Yukawa coupling.

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1 Introduction.

The pattern of fermion masses, their mixing, and the family replication, remain as the most outstanding problems of nowadays particle physics. The successful Standard Model (SM) based on the gauge group SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y$ can tolerate, but not explain the experimental results. Two main features that a consistent family theory should provide are
(i)-Within each charge sector, the masses increase with family by large factors

$$m_u << m_c << m_t, \quad m_d << m_s << m_b, \quad m_e << m_\mu << m_\tau,$$

(ii)-Within each family, the masses are quite different. In particular, for the third family we have

$$m_b << m_t.$$

The horizontal survival hypothesis[1] was invented in order to cope with this hierarchy without putting it by hand in the Yukawa couplings. According to this hypothesis, a certain symmetry should guarantee that at the unification scale all the Yukawa terms, with Yukawa couplings $y_{ff'}$, where f and f′ are flavor labels, vanish except for those corresponding to the third family for which $y_{tt} \sim y_{bb} \sim y_{\tau\tau}$. A different starting point was introduced in Ref. [2], under the name of modified survival hypothesis, demanding that all the Yukawa terms vanish at the unification scale except the diagonal one of the top quark, $y_{tt} \neq 0$.

In this paper we classify the continuous anomaly free horizontal symmetries that lead to the modified survival hypothesis. We take the number of families to be three and extend the SM group to SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y\otimes$G$_H$ and allow G$_H$ to be any of the subgroups of the most general family symmetry which commutes with SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y$. We keep the number of ingredients and parameters down to the minimum possible assuming that the model does not contain exotic fermions, with the only exception of a possible right-handed neutrino state for each family, and assuming that the Higgs fields transform as an irreducible representation of SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y\otimes$G$_H$

The most general family symmetry which commutes with SU(3)$_c\otimes$SU(2)$_L$ is

$$G = \mathrm{U}(3)_q\otimes\mathrm{U}(3)_u\otimes\mathrm{U}(3)_d\otimes\mathrm{U}(3)_l\otimes\mathrm{U}(3)_e\otimes\mathrm{U}(3)_\nu$$

where each factor is defined in the space of vectors $\eta = (\eta_1,\ \eta_2\ \eta_3)$ with $\eta = q,\ u,\ d,\ l,\ e$, or ν,

$$q = \left(\begin{array}{c} u \\ d \end{array} \right)_{\alpha L},\ u = u^c_{\alpha L},\ d = d^c_{\alpha L},\ l = \left(\begin{array}{c} \nu \\ e \end{array} \right)_L,\ e = e^c_L,\ \nu = \nu^c_L,$$

and where c denotes a charge conjugated field and α is a color index which will not be displayed in what follows. In what follows we also omit the helicity index L. Each $\mathrm{U}(3)_\eta = \mathrm{SU}(3)_\eta \otimes \mathrm{U}(1)_\eta$ contains a family independent subgroup $\mathrm{U}(1)_\eta$ and the SM $\mathrm{U}(1)_Y$ factor is contained in $\mathrm{U}(1)_q \otimes \mathrm{U}(1)_u \otimes \mathrm{U}(1)_d \otimes \mathrm{U}(1)_l \otimes \mathrm{U}(1)_e \otimes \mathrm{U}(1)_\nu$.

Obviously G is not itself a candidate for a gauged family symmetry since none of its $\mathrm{SU}(3)_\eta$ factors, with just one triplet of fermions, is anomaly free. In what follows we analyze all the continuous subgroups G_H of G which are anomaly free and which allow a (mass generating) Yukawa coupling only for the top quark. By mass generating Yukawa coupling we mean the coupling to fermions of the Higgs that develops an $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ breaking vacuum expectation value (VEV).

In section 2 we review and extend our previous results[3] for the simplest form of G_H, namely $\mathrm{G}_H = \mathrm{U}(1)_H$. In the following sections we increase the complexity of G_H. We find two models which satisfy our conditions. They can be seen either as $\mathrm{U}(1)_H$ models or as $\mathrm{U}(1)_{H1} \otimes \mathrm{U}(1)_{H2}$ models. In section 9 we make a brief analysis of the phenomenological implications of these two models. In section 10 we summarize our conclusions.

2 $\mathbf{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H}$ as an anomaly-free model.

A gauge theory is renormalizable if it is free of anomalies[4]. For this particular model only the gravitational[5] and chiral anomalies[4] are present[‡]. We demand cancellation of these anomalies by the power counting method. (The alternative of canceling the anomalies by the Green-Schwarz mechanism[7] requires additional assumptions at the string theory level[8].)

There are two different ways of canceling the anomalies. One is demanding cancellation of the anomalies within each family and the other one is canceling the anomalies among families.

[‡]The inclusion of gravitational constraints has successfully lead to the rederivation of some aspects of the Standard Model. See Re. [6] and references quoted therein.

2.1 Cancellation of anomalies within each family.

Assuming that there are no right-handed neutrinos, using the U(1) charges in Table 1a, and demanding freedom from chiral anomalies for $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_H$, we get the following set of equations ($i = 1, 2, 3$) for the $\mathrm{U}(1)_H$ hypercharges:

$$\begin{aligned}
[\mathrm{SU}(2)_L]^2 U(1)_H : &\quad Y_{li} + 3Y_{qi} = 0, &(1)\\
[\mathrm{SU}(3)_c]^2 U(1)_H : &\quad 2Y_{qi} + Y_{ui} + Y_{di} = 0, &(2)\\
[U(1)_Y]^2 U(1)_H : &\quad 2Y_{li} + 4Y_{ei} + \frac{2}{3}Y_{qi} + \frac{16}{3}Y_{ui} + \frac{4}{3}Y_{di} = 0, &(3)\\
U(1)_Y [U(1)_H]^2 : &\quad -Y_{li}^2 + Y_{ei}^2 + Y_{qi}^2 - 2Y_{ui}^2 + Y_{di}^2 = 0, &(4)\\
[\mathrm{grav}]^2 U(1)_H : &\quad 2Y_{li} + Y_{ei} + 3(2Y_{qi} + Y_{ui} + Y_{di}) = 0, &(5)\\
[U(1)_H]^3 : &\quad 2Y_{li}^3 + Y_{ei}^3 + 6Y_{qi}^3 + 3Y_{ui}^3 + 3Y_{di}^3 = 0, &(6)
\end{aligned}$$

For a Higgs field with $\mathrm{U}(1)_H$ charge Y_ϕ, a Yukawa coupling for the top quark is allowed if

$$Y_{q_3} + Y_{u3} = Y_\phi \tag{7}$$

whereas a bottom quark coupling is forbidden if

$$Y_{q_3} + Y_{d3} \neq -Y_\phi = Y_{\phi^\star}. \tag{8}$$

Eqs. 7 and 8 are however in contradiction with Eq. 2. Therefore, if a top quark mass arises at tree level, a bottom mass arises as well at the same level.

Including right handed neutrinos ν_i^c does not change this conclusion since Eq. 2 stays valid. The only changes are in Eqs. 5 and 6 which are now replaced by

$$\begin{aligned}
[\mathrm{grav}]^2 U(1)_H &: \; 2Y_{li} + Y_{ei} + Y_{\nu i} + 3(2Y_{qi} + Y_{ui} + Y_{di}) = 0, &(9)\\
[U(1)_H]^3 &: \; 2Y_{li}^3 + Y_{ei}^3 + 6Y_{qi}^3 + 3Y_{ui}^3 + 3Y_{di}^3 + Y_{\nu i}^3 = 0. &(10)
\end{aligned}$$

2.2 Cancellation of anomalies among families.

If the $\mathrm{U}(1)_H$ anomalies are canceled by an interplay among families, Eqs. 1 to 6 should be understood with a sum over $i = 1, 2, 3$. Eq. 4 then reads

$$\sum_i (-Y_{li}^2 + Y_{ei}^2 + Y_{qi}^2 - 2Y_{ui}^2 + Y_{di}^2) = 0. \tag{11}$$

A general class of solutions to the new anomaly cancellation equations which are linear in Y_H is characterized by the constraints

$$\sum_{i=1}^{3} Y_{\eta i} = 0 \tag{12}$$

for $\eta = q, u, d, l, e$. We will limit ourselves to this type of solutions, and within this set we will consider only those for which the u_i and l_i $\mathrm{U}(1)_H$ hypercharges are fixed to satisfy either

$$\begin{aligned} Y_{l1} = \delta_1 \equiv \delta, \;\; Y_{l2} = \delta_2 = -\delta, \;\; Y_{l3} = \delta_3 = 0, \\ Y_{u1} = \delta'_1 \equiv \delta', \;\; Y_{u2} = \delta'_2 = -\delta', \;\; Y_{u3} = \delta'_3 = 0, \end{aligned} \tag{13}$$

or any set of relations obtained by a permutation of the indices $i = 1, 2, 3$. (This guarantees that the ratios of $\mathrm{U}(1)_H$ hypercharges within the set of fermions of a given charge are rational numbers. For solutions with irrational numbers see Appendix A.) These solutions can be divided into four classes according to the way the cancellation occurs in Eq. 11.

<u>CLASS A</u> Lepton sector independent of quark sector.
$Y_{ei} = Y_{li} = \delta_i$ and $Y_{qi} = Y_{ui} = Y_{di} = \delta'_i, i = 1, 2, 3$. A model with a tree-level top quark mass arises if $Y_\phi = Y_{qi} + Y_{uj}$ for some i and j. There are five different models in this class characterized by $\mathrm{Y}_\phi = \pm 2\delta', \pm\delta'$ and 0 respectively. Simultaneously a tree-level bottom mass arises if there exist family indices k and l for which $Y_{qk} + Y_{dl} = -Y_\phi$. For example, if $Y_\phi = 2\delta'$ then $i = j = 1$ and $k = l = 2$ satisfy both conditions; this is signaled in Table 2 by the entries $(1, 1)_u$ and $(2, 2)_d$ in the Class A column and the $2\delta'$ row. The fact that in Table 2 there is at least one d-type entry for every u-type one for all the five models of Class A means that none of them is viable. (This stems from the equality $Y_{di} = Y_{ui}$ in Class A).

<u>CLASS B</u> Doublets independent of singles.
$Y_{qi} = Y_{li} = \delta_i$ and $Y_{ui} = Y_{di} = Y_{ei} = \delta'_i, i = 1, 2, 3$. There are nine different models in this class characterized by $\mathrm{Y}_\phi = \delta \pm \delta'$, $-(\delta \pm \delta')$, $\pm\delta$, $\pm\delta'$, and 0 respectively. Again none of these models is viable ($Y_{di} = Y_{ui}$ in Class B also).

<u>CLASS C</u> Interplay between doublet leptons and singlet quarks.
$Y_{di} = Y_{li} = \delta_i$ and $Y_{qi} = Y_{ui} = Y_{ei} = \delta'_i$, $i = 1, 2, 3$. There are now eleven different models in this class characterized by $\mathrm{Y}_\phi = \pm 2\delta'$, $\delta \pm \delta'$, $-(\delta \pm \delta')$, $\pm\delta$, $\pm\delta'$ and 0. As can be seen from Table 2, for $\delta \neq 0, \pm\delta', \pm 2\delta', \pm 3\delta'$ and $\delta' \neq 0$ there are two models in

which only one u-type mass and none d-type one develops at tree-level. These models are:
MI. Where the Higgs field has (U(1)$_Y$,U(1)$_H$) hypercharges equal to $(-1, 2\delta')$.
MII. Where the Higgs field has (U(1)$_Y$,U(1)$_H$) hypercharges equal to $(-1, -2\delta')$.
The rest of the models in this class are non-viable because a bottom quark mass arises at tree level. The U(1)$_H$ quantum numbers of the Yukawa terms in the models MI and MII are displayed in Appendix B.

CLASS D Same U(1)$_H$ hypercharge for the whole family.
This is a particular case of classes A,B, and C above, for which $\delta_i = \delta'_i$, which in turn implies[9] $Y_{qi} = Y_{ui} = Y_{di} = Y_{li} = Y_{ei} = \delta_i$. As far as the quark mass spectrum is concerned this class is equivalent to class A.

We may include right-handed neutrino fields within the above scheme either by setting $Y_{\nu 1} = -Y_{\nu 2} = \delta$, $Y_{\nu 3} = 0$ (or permutations of the indices 1,2,3), or by imposing $Y_{\nu i} = 0$ (which is one of the ingredients for the see-saw mechanism[10]).

3 SU(3)$_c$⊗SU(2)$_L$⊗U(1)$_Y$⊗U(1)$_{H1}$⊗ U(1)$_{H2}$.

Let us consider first the case where U(1)$_{H1}$ =U(1)^L and U(1)$_{H2}$ =U(1)^R. There are again two different ways of canceling the anomalies, the cancellation within each family, and the cancellation among families.

3.1 Cancellation of anomalies within each family.

Without right-handed neutrinos, the only anomaly-free solution corresponds to the trivial one Y^L=Y^R = 0. Let us see why.

The [SU(3)$_c$]^{2}U(1)$_{HL}$ constraint is Y$^L_{qi}$ = 0. The [SU(2)$_L$]^{2}U(1)^L constraint is Y$^L_{li}$ + 3Y$^L_{qi}$ = 0, which combined with the previous result implies Y$^L_{li}$ = 0. Hence, Y^L = 0 as stated. (This result still holds when we include right-handed neutrinos which are singlets under SU(2)$_L$ and SU(3)$_c$). Now the [SU(3)$_c$]^{2}U(1)^R constraint implies Y$^R_{ui}$ + Y$^R_{di}$ = 0. The [grav]^{2}U(1)^R constraint implies Y$^R_{ei}$ + 3Y$^R_{ui}$ + 3Y$^R_{di}$ = 0, which combined with the [SU(3)$_c$]^{2}U(1)^R constraint gives Y$^R_{ei}$ = 0. Now if we combine the last result with the [U(1)^R]^{2}U(1)$_Y$ constraint $(Y^R_{ei})^2 - 2(Y^R_{ui})^2 + (Y^R_{di})^2 = 0$, we get Y$^R_{di}$=Y$_{ui}$=Y$^R_{ei}$=0 as anticipated above.

When we include the right-handed neutrinos we still have $\mathrm{Y}^L = 0$ as commented in the previous paragraph. The anomaly cancellation equations for Y^R are then given by equations 1 - 4 and 9 and 10 with $Y_{li} = Y_{qi} = 0$. The solution to those equations is $Y^R_{ui} = Y^R_{\nu i} = -Y^R_{di} = -Y^R_{ei} = \kappa_i$, where κ_i are three arbitrary numbers. Therefore a Higgs field with $Y^R_\phi = \kappa_3$ will produce Dirac masses for the top and bottom quarks, and for the τ and ν_τ leptons as well.

3.2 Cancellation of anomalies among families.

Using the $\mathrm{U}(1)^L$ and $\mathrm{U}(1)^R$ charges in Table 1a we get the following set of equations to be satisfied simultaneously:

$$
\begin{array}{lll}
[\mathrm{SU}(2)_L]^2 U(1)^{L(R)}: & \sum_i (Y^L_{li} + 3Y^L_{qi}) = 0 & \\
[\mathrm{SU}(3)_c]^2 U(1)^{L(R)}: & \sum_i Y^L_{qi} = 0 & \sum_i (Y^R_{ui} + Y^R_{di}) = 0 \\
[U(1)_Y]^2 U(1)^{L(R)}: & \sum_i (3Y^L_{li} + Y^L_{qi}) = 0 & \sum_i (3Y^R_{ei} + 4Y^R_{ui} + Y^R_{di}) = 0 \\
U(1)_Y [U(1)^{L(R)}]^2: & \sum_i [(Y^L_{qi})^2 - (Y^L_{li})^2] = 0 & \sum_i [(Y^R_{ei})^2 - 2(Y^R_{ui})^2 + (Y^R_{di})^2] = 0 \\
[\mathrm{grav}]^2 U(1)^{L(R)}: & \sum_i (3Y^L_{qi} + Y^L_{li}) = 0 & \sum_i (Y^R_{ei} + 3Y^R_{ui} + 3Y^R_{di}) = 0 \\
[\mathrm{U}(1)^{L(R)}]^3: & \sum_i [3(Y^L_{qi})^3 + (Y^L_{li})^3] = 0 & \sum_i [(Y^R_{ei})^3 + 3(Y^R_{ui})^3 + 3(Y^R_{di})^3] = 0.
\end{array}
$$

The mixed anomalies related to $\mathrm{U}(1)^L[\mathrm{U}(1)^R]^2$ and $\mathrm{U}(1)^R[\mathrm{U}(1)^L]^2$ trivially vanish due to the fact that for every SM multiplet one of the two hypercharges Y^L or Y^R is always zero (see Table 1a).

Contrary to the case with a single $\mathrm{U}(1)_H$ factor group, the constraint given by eq. (12), follows fron the above anomaly cancellation eqs. Furthermore, the quadratic anomaly cancellation eqs. demand now that the case of cancellation of anomalies among doublets be independent of the cancellation among singlets. Therefore we set

$$Y^L_{l1} = \delta_1 \equiv \delta, \;\; Y^L_{l2} = \delta_2 = -\delta, \;\; Y^L_{l3} = \delta_3 = 0, \;\; Y^L_{qi} = Y^L_{li},$$

$$Y^R_{u1} = \delta'_1 \equiv \delta', \;\; Y^R_{u2} = \delta'_2 = -\delta', \;\; Y^R_{u3} = \delta'_3 = 0, \;\; Y^R_{ei} = Y^R_{di} = Y^R_{ui} = \delta'_i,$$

or any set of relations obtained from them by a permutation of the indices $i = 1, 2, 3$ (for other solutions see Appendix A). These solutions are similar to the solutions in Class B in section 2.2, hence the same conclusions follow.

3.3 $\mathbf{U(1)}_{H1} = \mathbf{U(1)}^{quarks}$, $\mathbf{U(1)}_{H2} = \mathbf{U(1)}^{leptons}$.

Again, if we demand cancellation of anomalies within each family, the linear constraints imply $\mathrm{Y}^{quarks} = 0$ for all fermions. When anomalies are canceled among families the situation is similar to that of class A models in the case $\mathrm{G}_H = \mathrm{U}(1)_H$. Setting

$$Y_{qi}^{quarks} = Y_{ui}^{quarks} = Y_{di}^{quarks} = \delta_i'$$

and

$$Y_{li}^{leptons} = Y_{ei}^{leptons} = \delta_i$$

we are led again to the conclusion that there are no viable models in this case.

3.4 $\mathbf{U(1)}_{H1}$ =$\mathbf{U(1)}^{que}$, $\mathbf{U(1)}_{H2}$ =$\mathbf{U(1)}^{dl}$.

Again, nontrivial horizontal hypercharges are compatible with anomaly cancellations only when these are realized among families.

Setting

$$Y_{qi}^{que} = Y_{ui}^{que} = Y_{ei}^{que} = \delta_i',$$

$$Y_{li}^{dl} = Y_{di}^{dl} = \delta_i,$$

we obtain the models MI and MII of section 2 with $\mathrm{Y}_\phi^{que} = \pm 2\delta'$, $Y_\phi^{dl} = 0$. For the Yukawa terms to be invariant under the full $\mathrm{G}_H = \mathrm{U}(1)^{que} \otimes \mathrm{U}(1)^{dl}$ a vanishing entry in the matrices of quantum numbers displayed in Appendix B should be obtained without the interplay of δ with δ'. Therefore the only condition that on δ' imposes the requirement that an invariant Yukawa term be allowed only for the top quark is just $\delta' \neq 0$.

We close this section by remarking that a glance to Eq. 11 should convince the reader that it is not viable to consider additional U(1) factors in G_H, $\mathrm{G}_H = \mathrm{U}(1)_{H1} \otimes$ $\mathrm{U}(1)_{H2} \otimes$ $\mathrm{U}(1)_{H3} \otimes$ ...That is, there is no other way of canceling the $\mathrm{U}(1)_Y[\mathrm{U}(1)_{H1}]^2$ anomalies.

4 $\mathbf{SU(3)}_c \otimes \mathbf{SU(2)}_L \otimes \mathbf{U(1)}_Y \otimes \mathbf{SU(2)}_H$.

All the possible models for this group have been exhaustively analyzed in Ref.[11]. We use here some of their results. The anomaly free $\mathrm{SU}(2)_H$ spectra in the absence of right-handed neutrinos shows that from the 9×27 possible arrangements of representations in the gauge group only 14 satisfy the chiral and global[12] anomaly constraints (there is not a gravitational anomaly for $\mathrm{SU}(2)_H$, and the global anomaly vanishes if the theory contains an even number of $\mathrm{SU}(2)_H$ doublets[12]). These fourteen possible models are presented in Eq.3 of Ref.[11] and analyzed in Sections. II, III, and in the

summary of the tree level results presented in Table IV of the same reference. From that Table (which is correct up to minor details) we see that none of the possible models is able to produce a tree level rank one mass matrix for the up quark sector simultaneously with a rank zero mass matrix for the down sector. In more detail the situation is the following:

From Eq. 3 in Ref.[11] we see that there are only 11 different quark arrangements to be considered in the 14 theories enumerated (Q_1, Q_8 and Q_{21} appear twice) where the possible quark $SU(2)_H$ representations $Q_i, i = 1-27$ are in Table 4a considering that in the table all the representations belong to the same horizontal group $SU(2)_H$. Now, $SU(2)_H$ does not act on the quark fields in Q_{21}, hence it has for the quark sector the same information than the SM has and we ignore it. We also ignore Q_{13} since in this arrangement $SU(2)_H$ does not act on the up quark sector. Then we should analyze only the following $SU(2)_H$ structures: Q_1, Q_4, Q_6, Q_8, Q_{10}, Q_{11}, Q_{12}, Q_{16}, and Q_{18}.

Since a mass term for the up quark sector is of the form $\langle\phi^\dagger\rangle u^c q$, and that for the down quark sector is of the form $\langle\phi'^\dagger\rangle d^c q$, and since $SU(2)_H$ is a (pseudo-) real group, it is obvious that when u^c and d^c are in the same representation of $SU(2)_H$, a Higgs field ϕ which produces a mass matrix for the up sector will produce, via $\phi^c = -\sigma_{2L}\phi^\star$ or $\phi^c = -\sigma_{2L}\sigma_{2H}\phi^\star$, the same mass matrix for the down sector (up to a Yukawa coupling constant). Therefore the structures Q_1, Q_4, Q_8, Q_{11}, Q_{12} and Q_{18} are not viable. We are then left only with the structures Q_6, Q_{10} and Q_{16} to be analyzed.

Q_{16} was analyzed in full detail in Ref.[11]. The $SU(3)_c\otimes SU(2)_H\otimes SU(2)_L\otimes U(1)_Y$ quantum numbers (QNs) for the quark sector in this model are $q \sim (3,2,2)_{1/3}\oplus(3,1,2)_{1/3}$, $u^c \sim (\overline{3},2,1)_{-4/3}\oplus(\overline{3},1,1)_{-4/3}$, $d^c \sim 3(\overline{3},1,1)_{2/3}$. An up quark mass term in Q_{16} has $SU(2)_H$ QNs according to the product $(2+1)\times(2+1) = 3+2(twice)+1(twice)$. Since a down quark mass term has QNs $(2+1)\times 1 = 2+1$, a Higgs field belonging to the representations 1 or 2 of $SU(2)_H$ will produce non zero mass matrices for the up and down sectors simultaneously. A Higgs field ϕ_k belonging to the representation 3 of $SU(2)_H$ generates a mass matrix only for the up quark sector of the form $\langle\phi_k^\dagger\rangle\sum_{c,b=1}^{2} u_a^c(\sigma^k)_{ab}q_b$, where each ϕ_k is an $SU(2)_L$ doublet with VEV of the form $(x_k, 0)^T$. With all[§] the $x_k \neq 0$ the rank of the up quark mass matrix is two, with $m_t = m_c$, which is unphysical[11].

[§]At the scale $\langle\phi_k\rangle$ $SU(2)_H$ is aready broken and we cannot use it to orientate $\langle\phi_k\rangle$ in the H space

Hence Q_{16} is not able to explain the known quark mass spectrum.

Now for Q_6, the $SU(3)_c \otimes SU(2)_H \otimes SU(2)_L \otimes U(1)_Y$ QNs for the quark sector are $q \sim (3,2,2)_{1/3} \oplus (3,1,2)_{1/3}$, $u^c \sim (\overline{3},2,1)_{-4/3} \oplus (\overline{3},1,1)_{-4/3}$, $d^c \sim (\overline{3},3,1)_{2/3}$. An up quark mass term has $SU(2)_H$ QNs 3, 2 or 1 as for Q_{16}, and a down quark mass term has $SU(2)_H$ QNs $(2+1)\times 3 = 4+2+3$. Hence, a Higgs field belonging to the representation 2 or 3 of $SU(2)_H$ produces masses both for the up and down quark sectors. A singlet Higgs field under $SU(2)_H$ generates a mass matrix only for the up sector, but it couples to the three families producing a rank three mass matrix and generating m_t, m_c and m_u at tree-level. Therefore Q_6 is discarded.

Finally, the $SU(3)_c \otimes SU(2)_H \otimes SU(2)_L \otimes U(1)_Y$ QNs for Q_{10} are $q \sim (3,3,2)_{1/3}$, $u^c \sim (\overline{3},3,1)_{-4/3}$, $d^c \sim 3(\overline{3},1,1)_{2/3}$. Now an up quark mass term has $SU(2)_H$ QNs $3\times 3 = 5+3+1$ and a down quark mass term is a triplet under $SU(2)_H$. Thus a Higgs field belonging to the representation 3 of $SU(2)_H$ will generate mass matrices for the up and down sectors at the same time, but a Higgs field in the representation 1 or 5 of $SU(2)_H$ will produce tree-level masses only for the up sector. However the mass matrices generated in both cases are rank three[11], producing three-level masses for the three families. Therefore this possibility, the last one in the case without right handed neutrinos, is also discarded.

When the right-handed neutrinos are included in the spectrum (one for each family), the number of models satisfying the anomaly constraints become 35 (see equation (35) in Ref.[11]). These 35 models correspond to the 14 structures of the neutrinoless case with the three neutrinos accommodated in the representation 3 of $SU(2)_H$, plus the same 14 structures of the neutrinoless case with the three neutrinos accommodated in the representation $1\oplus 1\oplus 1$ of $SU(2)_H$, plus seven more structures with the neutrinos accommodated in the representation (2+1) of $SU(2)_H$, and the quark spectrum given by (Q_1, Q_3, Q_5, Q_8, Q_{17}, Q_{19}, and Q_{21}). The structures Q_1, Q_8, and Q_{21} where analyzed and excluded in the previous paragraphs. Also Q_5 and Q_{17} are such that u^c and d^c are in the same $SU(2)_H$ representation, and in Q_{19} $SU(2)_H$ does not act on the up quark sector at all. We are thus left only with Q_3 to be analyzed here.

The $SU(3)_c \otimes SU(2)_H \otimes SU(2)_L \otimes U(1)_Y$ QNs for the quark sector in Q_3 are $q \sim (3,3,2)_{1/3}$, $u^c \sim (\overline{3},3,1)_{-4/3}$, $d^c \sim (\overline{3},2,1)_{2/3} \oplus (\overline{3},1,1)_{2/3}$. An up quark mass term has $SU(2)_H$ QNs 5,3,1 as in Q_{10}, and a down quark mass term is of the form $3\times(2+1) \sim 4+2+3$. A Higgs field

belonging to representation 3 of $SU(2)_H$ generates thus masses for the up and down quark sectors simultaneously. A Higgs field in a 1 or 5 representation of $SU(2)_H$ generates a mass matrix only for the up quark sector, but of rank 3 [11]. Therefore Q_3 is also ruled out.

The conclusion is that $SU(3)_c \otimes SU(2)_H \otimes SU(2)_L \otimes U(1)_Y$ can not explain why $m_t >> m_b$.

5 $\mathbf{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SU(2)_{HL} \otimes SU(2)_{HR}}$.

Gauge anomaly cancellation is satisfied if the $[SU(2)_{HL}]^2 U(1)_Y$ and $[SU(2)_{HR}]^2 U(1)_Y$ anomalies vanish while global anomaly cancellation requires that there be an even number of $SU(2)_{HL}$ and of $SU(2)_{HR}$ doublets. Tables 3 and 4a show the values of the anomalies for the various possible ways of assigning representations to the particle types.

Comparing the values from these tables, and without including right-handed neutrinos, only six theories satisfying gauge and global anomaly cancellation appear. They are: (L_1+Q_1, L_3+Q_4, L_5+Q_{12}, L_6+Q_{11}, L_7+Q_{18}, L_9+Q_{21}). The last of these models is just the SM and therefore $SU(2)_{HL} \otimes SU(2)_{HR}$ does not act upon it. Notice also from Table 4a that for the other five quark structures Q_1, Q_4, Q_{11}, Q_{12} and Q_{18} (and also for Q_{21} if we wish) the up and down sectors belong to the same $SU(2)_{HR}$ representation, and then, a Higgs field which produces a non-zero mass matrix for the up sector will produce a non-zero mass matrix for the down sector as well. Therefore, all of them are ruled out.

When we include the three right-handed neutrinos we see that they do not contribute to the gauge anomaly ($Y_{SM} = 0$ for ν_i^c), and their contribution to the global anomaly will also be zero if the neutrinos transform either as three singlets or as a triplet under $SU(2)_{HR}$ (N_1 and N_3 respectively in what follows). Now, when they transform as a singlet plus a doublet (N_2), they contribute with an extra doublet to the global anomaly. There are then a total of 14 models without anomalies, the six structures of the neutrinoless case each one added with N_1 or with N_3, plus the following two new structures: (L_4 + Q_8 + N_2) and (L_8 + Q_{17} + N_2). The first 12 models were already ruled out.

The two new quark structures Q_8 and Q_{17} are also such that the up and down quark sectors belong to the same representation of $SU(2)_{HR}$ (2+1). Therefore they are also ruled out. Our conclusion

is that SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y\otimes$ SU(2)$_{HL}\otimes$SU(2)$_{HR}$ is not able to explain why $m_t >> m_b$.

6 SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y\otimes$ SU(2)$_{H1}\otimes$SU(2)$_{H2}$.

In section 3.4 it was shown that U(1)$^{que}\otimes$U(1)^{dl} satisfies our requirements. We may then ask whether a different assignment of the η fields to SU(2)$_{H1}$ and SU(2)$_{H2}$, other than separating left-handed fields from right-handed ones, would lead to a viable model.

We start by looking for the representations content of the q, u and Higgs fields that give rise to a rank-one up-quark mass matrix. Let then $n_\eta = (1+1+1)$, (2+1), or 3 be the representation content of $\eta = q$ or u under some fixed SU(2)$_{H1}$ group. There are nine possibilities for $n = (n_q,\, n_u)$. Although the rank of the up quark mass matrices that are allowed in each case has been already discussed in the previous section, we follow here a slightly different path in order to be able to leave unspecified the SU(2)$_{H1}$ group. It is straightforward to compute the rank of the up-quark mass matrices in each case but we can also read it from the tables IV and V of Ref. [11]. The results are listed in table 4b together with a Q$_i$ arrangement which contains the same $(n_q,\, n_u)$ entry and with the Higgs representations that leads to a non zero rank.

After we discard from table 4b the cases with rank > 1, we are left with 8 $(n_q,\, n_u,\, n_\phi) = n_A$ arrangements. Given n_q and n_ϕ a down-quark mass term is forbidden if n_ϕ is not in the set of irreducible representations in the complete reduction of $n_q \times n_d$. Table 4c lists the above mentioned n_A together with the values of n_d which forbid a down-quark mass term and with the corresponding [SU(2)$_{H1}$]^{2}U(1)$_Y$ anomalies. As we can see, all the cases are anomalous and we have to include some leptons in SU(2)$_{H1}$. The contribution to the anomaly from $n_l = (1+1+1+)$, 2+1, 3 is, however, = 0, -2, -8 while that of $n_e = (1+1+1+)$, 2+1, 3 is 0, 2, 8 and that of $n_\nu = 0$. Therefore it is impossible to cancel the anomalies listed in table 4c. and none of the SU(2)$_{H1}\otimes$SU(2)$_{H2}$ models is viable. The same reasoning applies to SU(2)$_{H1}\otimes$U(1)$_{H1}\otimes$SU(2)$_{H2}\otimes$U(2)$_{H2}$ or any other G$_H$ such that SU(2)$_{H1}\otimes$SU(2)$_{H2} \subset$ G$_H \subset$ G. For the sake of illustration and completeness we discuss in the following two sections two cases with other higher symmetry G$_H$ groups.

7 $\mathbf{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes\ SU(3)_H}$.

This model with quarks and leptons in the vectorlike $3+\bar{3}$ representation of $SU(3)_H$ was introduced for the first time in the literature in Ref.[13] and analyzed later in Refs.[14]. In Ref.[15] the known quarks and leptons where assigned to the chiral 3+3 representation of $SU(3)_H$ canceling the anomalies with mirror fermions, and recently in Ref.[16], the anomalies in this last representation were canceled in a more general way.

[$SU(3)_c$, $SU(2)_L$] multiplets may belong to the 1, 3 or $\bar{3}$ representation of $SU(3)_H$. On the other hand, since SU(3) is not a real group, the cancellation of the $[SU(3)_H]^3$ and $[SU(3)_H]^2U(1)_Y$ anomalies is nontrivial; in particular the cancellation of the first anomaly is achieved only when the number of $SU(3)_H$ triplets equals the number of antitriplets. Finally, since there are 18 different quark fields (36 Weyl states) but only 6 different lepton fields (12 or 9 Weyl states depending on whether we include or not the right-handed neutrino states), then it is not always possible to cancel the quark anomalies with the lepton anomalies.

In Table 5 we include all the possible $SU(3)_H$ fermion field representation assignments which are free of the $SU(3)_H$ gauge anomalies, together with their $[SU(3)_H]^2U(1)_Y$ anomaly value. From this table we find that only the models M_3, M_4, M_8, M_{13}, M_{14}, M_{17}, M_{18}, and M_{19}, are safe. (Notice that without right-handed neutrino states, the only anomaly free model is M_4 and that for M_{13} and M_{14} $SU(3)_H$ does not act on the left-handed fields at all). M_{19}, M_3 and M_8 are not adequate candidates since M_{19} is just the SM, $SU(3)_H$ does not act on the quark sector of M_3, and $SU(3)_H$ does not act on the up quark sector of M_8.

In M_4, a mass term for the up sector has $SU(3)_H$ QNs $3\times\bar{3} = 8+1$, and a mass term for the down sector has the same $SU(3)_H$ QNs. Since both representations 8 and 1 are real, a Higgs field ϕ which produces a mass matrix for the up quark sector will also produce a down quark mass matrix via $i\sigma_{2L}\phi^\star$. This argument is also valid for M_{17} and M_{18} (see Refs.[13, 14]). Therefore M_4, M_{17} and M_{18} can not explain why $m_b << m_t$.

Now, for M_{13} and M_{14} a Higgs field ϕ' able to produce a mass term for the up sector $\langle\phi'\rangle^\dagger u^c q$ must be in the representation $\bar{3}$ of $SU(3)_H$. But such a Higgs field automatically produces a down quark mass term $\langle\widetilde{\phi}'\rangle^\dagger d^c q$ via $\widetilde{\phi}' \equiv i\sigma_{2L}\phi'^\star$. Thus again we conclude that $SU(3)_H$

by itself can not explain why $m_b << m_t$.

8 $\mathbf{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes\ SU(3)_{HL} \otimes SU(3)_{HR}}$.

Since $SU(3)_{HL}$ would act only on q_i and l_i, the $SU(3)_{HL}$ safe representations are only those for which the anomalies in the quark sector cancel exactly the anomalies in the lepton sector, which is impossible when q and/or l are in a nontrivial representation (3 or $\overline{3}$) due to the fact that in this case there would be six $SU(3)_L$ representations in q (due to color) while only two in l. Therefore $SU(3)_{HL} \otimes SU(3)_{HR}$ is equivalent to a single $SU(3)_H$ which accommodates the left-handed fields q_i and l_i in the representation 1+1+1. This information can be extracted from Table 5 identifying $SU(3)_H$ as $SU(3)_{HR}$.

From Table 5 we read that when q_i and l_i are in the representation 1+1+1 of $SU(3)_{HL}$ only M_{13} and M_{14} are anomaly free. But those two models are also ruled out by the same reason that they were ruled out in the previous section. Hence $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes$ $SU(3)_{HL} \otimes SU(3)_{HR}$ by itself can not explain why $m_b << m_t$.

9 Brief analysis of MI and MII models.

In the previous sections we have analyzed all the possible horizontal gauge models able to accommodate three families. From them we selected those models able to generate a rank one tree-level mass matrix for the up quark sector and a rank zero tree-level mass matrix for the down quark sector. We have found only two candidates named MI and MII, both of them with the horizontal structure $U(1)_H$ or $U(1)^{que} \otimes U(1)^{dl}$.

Table Ib depicts explicitly the $U(1)_H$ (or $U(1)^{que} \otimes U(1)^{dl}$) hypercharges of fermions in these models. These hypercharges are then responsible for the quantum numbers of all possible (mass generating) Yukawa terms displayed in Appendix B. As can be seen, only one Yukawa term has zero horizontal hypercharge,

$$\mathrm{MI}: \quad \phi^\dagger u_1^c \begin{pmatrix} u \\ d \end{pmatrix}_1$$

$$\mathrm{MII}: \quad \phi^\dagger u_2^c \begin{pmatrix} u \\ d \end{pmatrix}_2 .$$

In what follows we briefly comment on some aspects of these models.

9.1 The symmetry breaking chain.

For simplicity, let us introduce two Higgs fields ϕ_H and ϕ_{SM}, both of them developing vacuum expectation values. $\langle\phi_H\rangle \sim M_H$ and $\langle\phi_{SM}\rangle \sim M_Z$, such that

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_H \xrightarrow{M_H} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

$$\xrightarrow{M_Z} SU(3)_c \otimes U(1)_Q,$$

where $G_H = \mathrm{U}(1)_H$ or $\mathrm{U}(1)^{que}\otimes\mathrm{U}(1)^{dl}$ and where $Q = T_{3L} + Y/2$.

The $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y\otimes\mathrm{U}(1)_H$ QNs for ϕ_H and ϕ_{SM} are $(1, 1, 0, \pm(3\delta' + \delta)/2)$ and $(1, 2, -1, \pm 2\delta')$ respectively, where the $\mathrm{U}(1)_H$ hypercharges are chosen for further purposes, and the upper and down signs are related to MI and MII respectively.

The required Higgs system in this section is minimal in the sense that there is a Higgs $\mathrm{SU}(2)_L$ doublet associated with the SM mass scale, and a singlet which provides the desired mass of the $\mathrm{U}(1)_H$ gauge boson Z′, heavy enough to avoid conflict with experiments.

9.2 Bottom quark mass.

To generate a bottom quark mass matrix different from zero in MI or MII further ingredients must be added to the models. For this purposes we introduce two new Higgs fields[17] $\phi^{(1)}$ and $\phi^{(2)}$ which do not develop VEVs and with QNs given by $\phi^{(1)}(3, 1, -2/3, \mp 2\delta')$ and $\phi^{(2)}(3^\star, 1, 2/3, \mp(\delta + \delta'))$. With these new Higgs fields other Yukawa terms are allowed in the Lagrangian,

$$\mathcal{L}' = \epsilon_{\alpha\beta\gamma}(h_1 q_1^\alpha \sigma_{2L} q_1^\beta \phi^{(1)\gamma} + h_2 u_1^{c\alpha} d_1^{c\beta} \phi^{(2)\gamma}) + h.c.,$$

where σ_{2L} is the $\mathrm{SU}(2)_L$ metric, h_1 and h_2 are Yukawa coupling constants, and α, β, and γ are $\mathrm{SU}(3)_c$ indices. Then there will be a one loop level contribution to a rank one finite, down quark mass matrix. In this loop the top quark mass will act as a seed and the loop will be completed with the propagators of the $\phi^{(1)}$ and $\phi^{(2)}$ fields mixed at tree-level by a term of the form $\phi^{(1)}\phi^{(2)}\langle\phi_H\rangle\langle\phi_H\rangle$.

9.3 Consistence with experimental constraints.

In MI and MII flavor changing neutral currents (FCNC) are mediated by the new gauge boson Z'[9, 3] and by the Higgs fields ϕ_H, $\phi^{(1)}$

and $\phi^{(2)}$. The FCNCs resulting from Z' can be suppressed by giving the new gauge boson a mass equal to or larger than 100 TeVs.

The FCNCs resulting from Higgs couplings can also be suppressed by giving the Higgs bosons sufficiently large masses. Naively, a mass of 100 TeVs for each one of those scalar fields will be consistent with the experimental constraints. Since the scalar sector introduced up to this point serves only as a starting point for more realistic mass generation schemes, we will not pursue any detailed Higgs phenomenology in the present paper.

9.4 Masses for the other fermions and Mixing angles.

Another concern of horizontal models of this kind is how to generate the radiative corrections that are assumed to provide the smaller masses and mixing angles in the models. For this purposes the *cascade* mechanism may be invoked. In this mechanism the light particles gain masses at various orders of perturbation theory from masses induced at the previous order of approximation. This mechanism requires the introduction of new scalars and is presented in detail in Refs.[17, 18].

An interesting feature of a particular version of the cascade mechanism[17] is that it couples the up quark to the strange and bottom quarks and couples the down quark to the charm and top quarks through the higher order corrections, thus providing a natural explanation for the observation $m_u < m_d$.

9.5 Embedding in a higher symmetry model.

In the two models under consideration the traces of the horizontal hypercharges vanish in the family basis. Therefore $U(1)_H$ or $U(1)^{que} \otimes U(1)^{dl}$ can be embedded into a simple or semisimple group. Rescaling the H hypercharges we can write the generators as

$$T_3 = diag(1, 0, -1)$$

or as

$$(1/2)\lambda_3 = diag(1/2, -1/2, 0)$$

corresponding to (one of) the diagonal generators of an $SU(2)_H$ or an $SU(3)_H$ group. The higher symmetry horizontal group can then be $SU(2)_H$, $SU(2)^{que} \otimes SU(2)^{dl}$, $SU(3)_H$, or $SU(3)^{que} \otimes SU(3)^{dl}$. In any of these cases the model should contain additional features, such

as extra fermions, in order to avoid the constraints that lead us to discard them.

10 Conclusion

The requirement of anomaly cancellation for the SM augmented with a horizontal factor, and with no additional fermions, other than right handed neutrinos, constitutes an strong condition. Without forcing by hand the orientation of the vacuum, the set of the viable G_H, groups is limited to $\mathrm{U}(1)_H$ and $\mathrm{U}(1)_{H1}\otimes\mathrm{U}(1)_{H2}$. In each case two models have been found with the fermions having the horizontal hypercharges displayed in table 1b.

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APPENDIX A. Irrational and complex solutions to the anomaly constraints

In this appendix we present a new set of solutions to the anomaly constraint equations for the gauge group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_H$, for the case when the anomalies are canceled by an interplay among families. We look for solutions to Eqs. 1 to 6 where a sum over $i = 1, 2, 3$ should be understood as indicated in section 2.2, and again we restrict to the case where $\sum_{i=1}^{3} \mathrm{Y}_{\eta i} = 0$. We classify this new set of solutions in the following classes:

<u>CLASS A′</u> With $\mathrm{Y}_{ui}=\mathrm{Y}_{di}=\mathrm{Y}_{qi} = 0; i = 1, 2, 3$
The constraint equations are now

$$\sum_{i=1}^{3} Y_{ei} = \sum_{i=1}^{3} Y_{li} = 0$$

$$\sum_{i=1}^{3} Y_{ei}^2 = \sum_{i=1}^{3} Y_{li}^2 \equiv 2a$$

$$\sum_{i=1}^{3} Y_{ei}^3 = -2\sum_{i=1}^{3} Y_{li}^3 \equiv -3b \tag{A1}$$

where a and b are arbitrary numbers. This class and class A of section 2 overlap when b= 0, a = δ^2, $\delta' = 0$.

The roots of the cubic equation

$$Y^3 - aY + b = 0$$

satisfy eqs. A1 and their ratios are irrational or complex numbers for b $\neq$ 0.

Other classes are given by:

<u>CLASS B′</u> With $\mathrm{Y}_{ei}=\mathrm{Y}_{ui}=\mathrm{Y}_{di}=0$; i=1,2,3

<u>CLASS C′</u> With $\mathrm{Y}_{qi}=\mathrm{Y}_{ui}=\mathrm{Y}_{ei}=0$; i=1,2,3, etc.

The exotic solutions presented in this Appendix are not considered in the main text because we do not envisage how to fit them in a natural way into more general theories.

APPENDIX B. Horizontal quantum numbers of Yukawa terms in the MI and MII Models.

I). MI model.

$$\mathcal{L}^{(\mathbf{2/3})} \equiv \phi^\dagger u_i^c \begin{pmatrix} u \\ d \end{pmatrix}_j : \quad \begin{pmatrix} 0 & -2\delta' & -\delta' \\ -2\delta' & -4\delta' & -3\delta' \\ -\delta' & -3\delta' & -2\delta' \end{pmatrix}$$

$$\mathcal{L}^{(\mathbf{-1/3})} \equiv \phi^{c\dagger} d_i^c \begin{pmatrix} u \\ d \end{pmatrix}_j : \quad \begin{pmatrix} \delta+3\delta' & \delta+\delta' & \delta+2\delta' \\ -\delta+3\delta' & -\delta+\delta' & -\delta+2\delta' \\ 3\delta' & \delta' & 2\delta' \end{pmatrix}$$

$$\mathcal{L}^{(\mathbf{-1})} \equiv \phi^{c\dagger} e_i^c \begin{pmatrix} \nu \\ e \end{pmatrix}_j : \quad \begin{pmatrix} \delta+3\delta' & -\delta+3\delta' & 3\delta' \\ \delta+\delta' & -\delta+\delta' & \delta' \\ \delta+2\delta' & -\delta+2\delta' & 2\delta' \end{pmatrix}$$

$$\mathcal{L}^{(\mathbf{0})} \equiv \phi^\dagger \nu_i^c \begin{pmatrix} \nu \\ e \end{pmatrix}_j \begin{cases} Y_{\nu i} = \delta_i : & \begin{pmatrix} 2\delta-2\delta' & -2\delta' & \delta-2\delta' \\ -2\delta' & -2\delta-2\delta' & -\delta-2\delta' \\ \delta-2\delta' & -\delta-2\delta' & -2\delta' \end{pmatrix} \\[2ex] Y_{\nu i} = 0 : & \begin{pmatrix} \delta-2\delta' & -\delta-2\delta' & -2\delta' \\ \delta-2\delta' & -\delta-2\delta' & -2\delta' \\ \delta-2\delta' & -\delta-2\delta' & -2\delta' \end{pmatrix} \end{cases}$$

II). MII model.

$$\mathcal{L}^{(\mathbf{2/3})} \equiv \phi^\dagger u_i^c \begin{pmatrix} u \\ d \end{pmatrix}_j : \quad \begin{pmatrix} 4\delta' & 2\delta' & 3\delta' \\ 2\delta' & 0 & \delta' \\ 3\delta' & \delta' & 2\delta' \end{pmatrix}$$

$$\mathcal{L}^{(\mathbf{-1/3})} \equiv \phi^{c\dagger} d_i^c \begin{pmatrix} u \\ d \end{pmatrix}_j : \quad \begin{pmatrix} \delta-\delta' & \delta-3\delta' & \delta-2\delta' \\ -\delta-\delta' & -\delta-3\delta' & -\delta-2\delta' \\ -\delta' & -3\delta' & -2\delta' \end{pmatrix}$$

$$\mathcal{L}^{(\mathbf{-1})} \equiv \phi^{c\dagger} e_i^c \begin{pmatrix} \nu \\ e \end{pmatrix}_j : \quad \begin{pmatrix} \delta - \delta' & -\delta - \delta' & -\delta' \\ \delta - 3\delta' & -\delta - 3\delta' & -3\delta' \\ \delta - 2\delta' & -\delta - 2\delta' & -2\delta' \end{pmatrix}$$

$$\mathcal{L}^{(\mathbf{0})} \equiv \phi^{\dagger} \nu_i^c \begin{pmatrix} \nu \\ e \end{pmatrix}_j \begin{cases} Y_{\nu i} = \delta_i : & \begin{pmatrix} 2\delta + 2\delta' & 2\delta' & \delta + 2\delta' \\ 2\delta' & -2\delta + 2\delta' & -\delta + 2\delta' \\ \delta + 2\delta' & -\delta + 2\delta' & 2\delta' \end{pmatrix} \\ \\ Y_{\nu i} = 0 : & \begin{pmatrix} \delta + 2\delta' & -\delta + 2\delta' & 2\delta' \\ \delta + 2\delta' & -\delta + 2\delta' & 2\delta' \\ \delta + 2\delta' & -\delta + 2\delta' & 2\delta' \end{pmatrix} \end{cases}$$

Here, as well as in the main text, the contraction of spinor indices should be understood taking into account the charge conjugation matrix C,

$$u^c q \equiv u_L^{cT} C q_L, \qquad C = i\gamma_2\gamma_0.$$

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Table 1a. $U(1)_Y$, $U(1)_H$, $U(1)^L$ and $U(1)^R$ charges for the known fermions. $i = 1, 2, 3$ is a flavor index denoting first, second and third family respectively. The Y_{SM} values stated are family independent. All fermions are left handed.

	$l_i = (\nu, e)_i$	e_i^c	$q_i = (\mathrm{u,d})_i$	u_i^c	d_i^c	ν_i^c
Y_{SM}	-1	2	1/3	$-4/3$	2/3	0
Y_H	Y_{li}	Y_{ei}	Y_{qi}	Y_{ui}	Y_{di}	$Y_{\nu i}$
Y^L	Y_{li}^L	0	Y_{qi}^L	0	0	0
Y^R	0	Y_{ei}^R	0	Y_{ui}^R	Y_{di}^R	$Y_{\nu i}^R$

Table 1b. $U(1)_Y$ and $U(1)^{que} \otimes U(1)^{dl}$ [or $U(1)_H$ with $Y_H = Y^{que} + Y^{dl}$] charges, as 3×3 diagonal matrices in the family basis, for the known fermions in MI and MII models. The Y_{SM} matrices are proportional to the 3×3 unit matrix.

	$U(1)^{que}$			$U(1)^{dl}$		
	$q_i = (u,\, d)_i$	u_i^c	e_i^c	$l_i = (\nu,\, e)_i$	d_i^c	ν_i^c
Y_{SM}	1/3	-4/3	2	-1	2/3	0
Y^{que}	(δ', -δ', 0)	(δ', -δ', 0)	(δ', -δ', 0)	0	0	0
Y^{dl}	0	0	0	(δ, -δ, 0)	(δ, -δ, 0)	(δ, -δ, 0) or 0
Y_H	(δ', -δ', 0)	(δ', -δ', 0)	(δ', -δ', 0)	(δ, -δ, 0)	(δ, -δ, 0)	(δ, -δ, 0) or 0

Table 2. Summary of tree-level mass terms for all the possible models with $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y\otimes\mathrm{U}(1)_H$ symmetry group allowed by the indicated Higgs field hypercharge Y_ϕ.

Y_ϕ	CLASS A	CLASS B	CLASS C
$2\delta'$	$(1{,}1)_u$;$(2{,}2)_d$		$(1{,}1)_u$
$-2\delta'$	$(2,2)_u$; $(1,1)_d$		$(2{,}2)_u$
0	$(1,2)_u$; $(2,1)_u(3,3)_u$; $(1,2)_d$; $(2,1)_d$; $(3,3)_d$	$(3,3)_u$; $(3,3)_d$	$(1,2)_u$; $(2,1)_u$; $(3,3)_u$; $(3,3)_d$
δ'	$(1,3)_u$; $(3,1)_u$; $(2,3)_d$; $(3,2)_d$	$(3,1)_u$; $(3,2)_d$	$(1,3)_u$; $(3,1)_u$; $(2,3)_d$
$-\delta'$	$(2,3)_u$; $(3,2)_u$; $(1,3)_d$; $(3,1)_d$	$(3,2)_u$; $(3,1)_d$	$(2,3)_u$; $(3,2)_u$; $(1,3)_d$
$\delta+\delta'$		$(1,1)_u$; $(2,2)_d$	$(2,2)_d$
$-\delta+\delta'$		$(2,1)_u$; $(1,2)_d$	$(2,1)_d$
$\delta-\delta'$		$(1,2)_u$; $(2,1)_d$	$(1,2)_d$
$-\delta-\delta'$		$(2,2)_u$; $(1,1)_d$	$(1,1)_d$
δ		$(1,3)_u$; $(2,3)_d$	$(3,1)_d$
$-\delta$		$(2,3)_u$; $(1,3)_d$	$(3,2)_d$

Table 3. Possible lepton $SU(2)_{HL}\otimes SU(2)_{HR}$ representation assignments, and their anomalies. $[HL(R)]^2Y$ stands for $[SU(2)_{HL(R)}]^2U(1)_Y$. The representation for l_i refers to the $SU(2)_{HL}$ group and the representation for e_i^c refers to the $SU(2)_{HR}$ group.

	l_i	e_i^c	$[HL]^2Y$. anomaly	$[HR]^2Y$. anomaly	$2_L^{\prime s}$	$2_R^{\prime s}$
L_1	3	3	-8	8	0	0
L_2	3	2+1	-8	2	0	1
L_3	2+1	3	-2	8	2	0
L_4	2+1	2+1	-2	2	2	1
L_5	3	1+1+1	-8	0	0	0
L_6	1+1+1	3	0	8	0	0
L_7	2+1	1+1+1	-2	0	2	0
L_8	1+1+1	2+1	0	2	0	1
L_9	1+1+1	1+1+1	0	0	0	0

Table 4a. Possible quark $SU(2)_{HL} \otimes SU(2)_{HR}$ representation assignments, and their anomalies. $[HL(R)]^2Y$ stands for $[SU(2)_{HL(R)}]^2U(1)_Y$. The representation for q_i refers to the $SU(2)_{HL}$ group and the representation for u_i^c and d_i^c refers to the $SU(2)_{HR}$ group.

	q_i	u_i^c	d_i^c	$[HL]^2Y$ anomaly	$[HR]^2Y$ anomaly	$2_L^{'s}$	$2_R^{'s}$
Q_1	3	3	3	8	-8	0	0
Q_2	3	2+1	3	8	4	0	3
Q_3	3	3	2+1	8	-14	0	3
Q_4	2+1	3	3	2	-8	6	0
Q_5	3	2+1	2+1	8	-2	0	6
Q_6	2+1	2+1	3	2	4	6	3
Q_7	2+1	3	2+1	2	-14	6	3
Q_8	2+1	2+1	2+1	2	-2	6	6
Q_9	3	1+1+1	3	8	8	0	0
Q_{10}	3	3	1+1+1	8	-16	0	0
Q_{11}	1+1+1	3	3	0	-8	0	0
Q_{12}	3	1+1+1	1+1+1	8	0	0	0
Q_{13}	1+1+1	1+1+1	3	0	8	0	0
Q_{14}	1+1+1	3	1+1+1	0	-16	0	0
Q_{15}	2+1	1+1+1	2+1	2	2	6	3
Q_{16}	2+1	2+1	1+1+1	2	-4	6	3
Q_{17}	1+1+1	2+1	2+1	0	-2	0	6
Q_{18}	2+1	1+1+1	1+1+1	2	0	6	0
Q_{19}	1+1+1	1+1+1	2+1	0	2	0	3
Q_{20}	1+1+1	2+1	1+1+1	0	-4	0	3
Q_{21}	1+1+1	1+1+1	1+1+1	0	0	0	0
Q_{22}	3	2+1	1+1+1	8	-4	0	3
Q_{23}	3	1+1+1	2+1	8	2	0	3
Q_{24}	2+1	3	1+1+1	2	-16	6	0
Q_{25}	2+1	1+1+1	3	2	8	6	0
Q_{26}	1+1+1	3	2+1	0	-14	0	3
Q_{27}	1+1+1	2+1	3	0	4	0	3

Table 4b. Rank of the up-quark mass matrix for the different representation contents of q, u and ϕ under some $\mathrm{SU}(2)_{H1}$ group. Q_i is one of the quark arrangements with the same n_q and n_u content which helps in finding the corresponding case in tables IV and V of Ref. [11].

n_q	n_u	Q_i	n_ϕ	rank
1+1+1	1+1+1	Q_{21}	1	3
1+1+1	2+1	Q_{17}	1	1
			2	1
1+1+1	3	Q_{11}	3	1
2+1	1+1+1	Q_{18}	1	1
			2	1
2+1	2+1	Q_{6}	1	3
			2	2
			3	2
2+1	3	Q_{4}	2	2
			3	1
			4	2
3	1+1+1	Q_{12}	3	1
3	2+1	Q_{5}	2	2
			3	1
			4	2
3	3	Q_{10}	1	3
			3	2
			5	3

Table 4c. Quark and Higgs $\mathrm{SU}(2)_{H1}$ representations with a rank-one up-quark mass matrix and with simultaneous rank-zero down-quark mass matrix and the corresponding $[\mathrm{SU}(2)_{H1}]^2\ \mathrm{U}(1)_Y$ anomaly.

n_q	n_u	n_d	n_ϕ	$[\mathrm{SU}(2)_{H1}]^2\ \mathrm{U}(1)_Y$
1+1+1	2+1	3	1	4
1+1+1	2+1	1+1+1	2	-4
1+1+1	2+1	3	2	4
1+1+1	3	1+1+1	3	-16
1+1+1	3	2+1	3	-14
2+1	1+1+1	3	1	10
2+1	3	1+1+1	3	-14

Table 5. Possible quark and lepton $SU(3)_H$ representations which are free of the gauge $SU(3)_H$ anomaly, where a 1 value stands for $1+1+1$. $[3H]^2Y$ stands for the $[SU(3)_H]^2U(1)_Y$ anomaly value. The number of representations can be doubled by the replacement $3 \leftrightarrow \bar{3}$, but the new 18 arrangements are equivalent to the present ones (as far as the low energy phenomenology is concerned).

	e_i^c	ν_i^c	l_i	u_i^c	d_i^c	q_i	$[3H]^2Y$
M_1	3	$\bar{3}$	1	1	1	1	2
M_2	1	1	1	3	$\bar{3}$	1	-2
M_3	3	3	$\bar{3}$	1	1	1	0
M_4	1	1	1	3	3	$\bar{3}$	0
M_5	3	1	3	$\bar{3}$	1	1	-4
M_6	3	1	3	1	$\bar{3}$	1	2
M_7	1	3	3	$\bar{3}$	1	1	-6
M_8	1	3	3	1	$\bar{3}$	1	0
M_9	3	1	3	3	1	$\bar{3}$	-2
M_{10}	3	1	3	1	3	$\bar{3}$	4
M_{11}	1	3	3	3	1	$\bar{3}$	-4
M_{12}	1	3	3	1	3	$\bar{3}$	2
M_{13}	3	$\bar{3}$	1	$\bar{3}$	3	1	0
M_{14}	$\bar{3}$	3	1	$\bar{3}$	3	1	0
M_{15}	$\bar{3}$	3	1	3	3	$\bar{3}$	2
M_{16}	3	$\bar{3}$	1	3	3	$\bar{3}$	2
M_{17}	3	3	$\bar{3}$	3	3	$\bar{3}$	0
M_{18}	$\bar{3}$	$\bar{3}$	3	3	3	$\bar{3}$	0
M_{19}	1	1	1	1	1	1	0